

**ON THE NULLSTELLENSÄTZE FOR STEIN SPACES
AND C -ANALYTIC SETS**

JOSÉ F. FERNANDO

(JOINT WORK WITH: FRANCESCA ACQUISTAPACE, FABRIZIO BROGLIA)

In this seminar we present the real Nullstellensatz for the ring $\mathcal{O}(X)$ of analytic functions on a C -analytic set $X \subset \mathbb{R}^n$ in terms of the *saturation* of Lojasiewicz's radical in $\mathcal{O}(X)$: *The ideal $\mathcal{I}(\mathcal{Z}(\mathfrak{a}))$ of the zero-set $\mathcal{Z}(\mathfrak{a})$ of an ideal \mathfrak{a} of $\mathcal{O}(X)$ coincides with the saturation $\widetilde{\sqrt{\mathfrak{a}}}$ of Lojasiewicz's radical $\sqrt{\mathfrak{a}}$.* If $\mathcal{Z}(\mathfrak{a})$ has 'good properties' concerning Hilbert's 17th Problem, then $\mathcal{I}(\mathcal{Z}(\mathfrak{a})) = \widetilde{\sqrt{\mathfrak{a}}}$ where $\sqrt{\mathfrak{a}}$ stands for the *real radical* of \mathfrak{a} . The same holds if we replace $\sqrt{\mathfrak{a}}$ with the *real-analytic radical* $\sqrt[\mathbb{R}]{\mathfrak{a}}$ of \mathfrak{a} , which is a natural generalization of the real radical ideal in the C -analytic setting. We also revisit the classical results concerning (Hilbert's) Nullstellensatz in the framework of (complex) Stein spaces.