

# SMI Course Kähler-Einstein metrics

## Book of abstracts

### Organizers

Daniele Angella (Università di Firenze)

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Palazzone della Scuola Normale Superiore, Cortona (AR)  
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## Participants

**Altavilla Amedeo** (Università Politecnica delle Marche)  
**Angella Daniele** (Università di Firenze)  
**Calamai Simone** (Università di Firenze)  
**Chen Gao** (Stony Brook University)  
**Collari Carlo** (Università di Firenze)  
**Gauduchon Paul** (Ecole Polytechnique)  
**Li Long** (Université de Grenoble)  
**Maccheroni Roberta** (Università di Parma)  
**Maschio Michele** (Università di Ferrara )  
**Morelato Andrea** (LMU Munich)  
**Otiman Alexandra** (University of Bucharest)  
**Pediconi Francesco** (Università di Firenze)  
**Pujia Mattia** (Università degli Studi di Torino)  
**Raffero Alberto** (Università di Parma)  
**Rossi Federico A.** (Università di Milano-Bicocca)  
**Scarpa Carlo** (SISSA)  
**Spotti Cristiano** (University of Aarhus)  
**Sroka Marcin** (Jagiellonian University)  
**Tadano Homare** (Osaka University)  
**Tardini Nicoletta** (Università di Pisa)  
**Thung Daniel** (Ludwig-Maximilian University Munich)  
**Ugolini Riccardo** (University of Ljubljana)  
**Xia Mingchen** (ENS paris)  
**Yao Chengjian** (Université Libre de Bruxelles)  
**Zedda Michela** (Università del Salento)  
**Zheng Kai** (University of Warwick)

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## Course

### Kähler-Einstein metrics

CRISTIANO SPOTTI

**Abstract.** Aim of the course is to present classical and recent results on Kaehler-Einstein metrics on compact complex manifolds, focusing on existence (e.g., Yau's proof of the Calabi's conjecture, alpha invariant criterion), obstructions, relations to algebraic geometric notions of stability (e.g. K-stability) and examples (e.g., KE metrics on Toric Fano manifolds).

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## Talks

### Classification of gravitational instantons

Tuesday 02, 16:00

GAO CHEN

**Abstract.** A gravitational instanton is a noncompact complete Calabi-Yau surface with faster than quadratic curvature decay. In this talk, I will discuss the classification of gravitational instantons. This is a joint work with X. X. Chen.

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### The convexity of the Ding functional

Wednesday 03, 14:30

PAUL GAUDUCHON

**Abstract.** TBA

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### Regularization of plurisubharmonic functions, revisited

Thursday 04, 16:00

LONG LI

**Abstract.** In this talk, we will present a new regularization technique of quasi-plurisubharmonic functions on a Kaehler manifold. It is a "discrete" or "localized" version of Demailly's regularization. The basic idea is to take convolutions of a plurisubharmonic function on a collection of coordinate balls first, and then we build a global object by glueing each piece together. Therefore, the advantage

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of this method is that the complex Hessians of the approximation functions have a better convergence behavior than previous results near the center of each ball in the collection. Moreover, all these centers form a  $\delta$ -net on the manifold eventually when the radius of the ball converges to zero.

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## Twisted cohomology of LCS solvmanifolds

Tuesday 02, 14:30

ALEXANDRA OTIMAN

**Abstract.** Locally conformally symplectic (LCS) manifolds provide a natural context for the study of Morse-Novikov cohomology (also known as twisted), which is the cohomology with values in a flat bundle. We present its various aspects and focus on the twisted cohomology of LCS solvmanifolds, of particular interest being the Inoue surfaces and Oeljeklaus-Toma manifolds. The talk is based on joint work with Daniele Angella and Nicoletta Tardini.

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## Hypersymplectic 4-manifolds, the $G_2$ Laplacian flow and extension under bounded torsion

Thursday 04, 14:30

CHENGJIAN YAO

**Abstract.** Hypersymplectic structure was introduced by Donaldson in his programme of studying the adiabatic limits of  $G_2$  manifolds. It is triple of symplectic forms on a differentiable 4-manifold, spanning a maximal positive subspace of  $\Lambda^2$  at each point. HyperKähler manifolds of dimension 4 give rich sources of hypersymplectic structures, and were conjectured by Donaldson to be the “only source” in the compact case. We study a geometric flow of such structures, designing to deform a given hypersymplectic structure to a hyperKähler one in the same cohomology class. We show that the flow does not develop finite time singularity with uniformly bounded torsion. Notice that our flow is a dimensional reduction of the more well-known  $G_2$  Laplacian flow introduced by Hitchin and Bryant to study the existence of metrics with  $G_2$  holonomy. This is a joint work with Joel Fine.

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## Convergence of the Calabi flow

Friday 05, 14:30

KAI ZHENG

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**Abstract.** We will present convergence theorems of the global solution to the Calabi flow on extremal Kähler surfaces. We will also discuss the regularity scales, which were introduced to obtain new estimates of the Calabi flow.

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## Timetable

	monday 02	tuesday 03	wednesday 04	thursday 05	friday 06
08:00 – 09:00	<i>breakfast</i>	<i>breakfast</i>	<i>breakfast</i>	<i>breakfast</i>	<i>breakfast</i>
09:00 – 10:30	Spotti 1	Spotti 3	Spotti 5	Spotti 7	Spotti 9
10:30 – 11:00	<i>coffee break</i>	<i>coffee break</i>	<i>coffee break</i>	<i>coffee break</i>	<i>coffee break</i>
11:00 – 12:30	Spotti 2	Spotti 4	Spotti 5	Spotti 8	Spotti 10
12:30 – 14:30	<i>lunch</i>	<i>lunch</i>	<i>lunch</i>	<i>lunch</i>	<i>lunch</i>
14:30 – 15:30		Otman	Gauduchon	Yao	Zheng
15:30 – 16:00		<i>coffee break</i>	<i>coffee break</i>	<i>coffee break</i>	<i>coffee break</i>
16:00 – 17:00		Chen	exercises and discussion	Li	exercises and discussion